pertaining to sight and vision, has been somewhat generalized. An analogous pair of definitions for *optical* would be that the dimensions of optical components are enormous compared with λ, or λ is of the order of a micron. Putting all these statements together, we have the comparative meaning

$L \sim \lambda$	$\lambda < L < 10^3\lambda$	$L > 10^3\lambda$
Microwave	Nanowave	Optical

and the definite meaning as shown in Fig. 1. It should be noted that *nanowave*, in its comparative meaning, is synonymous with *quasioptic* (Latin prefix, Greek stem!).

I hope that the new word *nanowave* will be given some serious consideration for use in this growing new technology where the traditional fields of optics and radio engineering meet and overlap. As an example of how it might be used, consider the title of the recent conference in Colorado entitled *Boulder Millimeter Wave and Far Infrared Conference*. It might have been called the *Boulder Nanowave Conference*. The conference announcement proclaims a special issue of the *IEEE Proceedings* to be titled "Millimeter Waves and Beyond." An alternate title could be "Infrared and Below," but why not "Nanowaves—a new Frontier"?

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Transverse Resonance Solution of Uniform Trapezoidal Waveguides

Waveguides of nonconventional cross sections are of interest, and several approximate techniques are being developed. Yashkin [1]–[3] approximated the complicated cross section of waveguides by a cluster of rectangles. For example, the lateral sides of a uniform isosceles trapezoidal waveguide were approximated [1] by three steps, as shown in Fig. 1, thus forming five rectangular guides. The solution of the wave equation was sought by matching the five separate solutions at the boundaries of the adjacent rectangular guides. This led to a system of transcendental equations, which is often laborious to solve. Yashkin's theoretical calculations are verified experimentally as shown in Table I.

The transverse resonance method [4] can easily be applied to the uniform trapezoidal waveguide with the three-step deformation in each lateral side. The cross-sectional view can be considered as cascaded transmission lines with short-circuited ends, thus forming a resonant cavity with propagation in the x direction. With this approximation, one transcendental equation is easily obtained and solved for the lowest cutoff wave number.

For simplicity, Yashkin let

$$s = a + 2\Delta, \quad d = 2\Delta, \quad a = \pi$$

and defined η as the ratio of the cutoff wave

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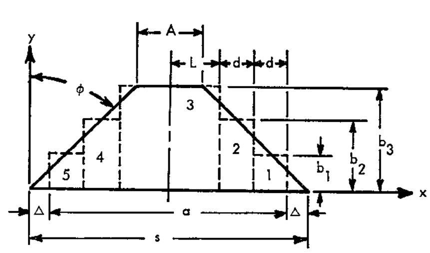

Fig. 1. Trapezoidal guide approximated by step deformations in lateral sides.

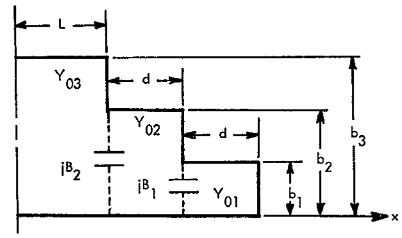

Fig. 2. Transmission line equivalent of one-half the symmetrical trapezoidal waveguide.

TABLE I
η CALCULATED FOR $b/s = 0.25$; WITH AND WITHOUT THE "DISCONTINUITY" ADMITTANCE

ϕ	Yashkin [1]		Transverse Resonance Method			
	theo. 3 steps	exp.	2 steps	3 steps	2 steps w/B	3 steps w/B's
0°	1.00	1 00	1.000	1.000	1.000	1.000
10°	1.04	1.04	1 046	1.046	1.046	1.045
20°	1.10	1.10	1.100	1.099	1.099	1.099
30°	1.17	1.17	1.167	1.169	1 164	1.164
40°	1.26	1.27	1.255	1.254	1.249	1.246
50°	1.33	1.33	1.389	1.383	1.360	1.356
60°	1.36	1.37	1.547	1.529	1.451	1.448

number of a rectangular waveguide to that of the uniform isosceles trapezoidal waveguide. That is,

$$\eta = \frac{2s}{2\pi/k} = \left(1 + \frac{2\Delta}{\pi}\right) k \qquad (1)$$

where k is the cutoff wave number of the trapezoidal guide.

For the guide shown in Fig. 1, the transverse resonance method dictates that the input admittance looking to the right or to the left from the center line is zero for TE (transverse electric) mode of propagation. A "discontinuity" admittance exists at every step as discussed by Whinnery and Jamieson [5]. When only a two-step approximation is considered, the transverse resonance method with the "discontinuity" admittance included in the transmission line equations yields the following simple transcendental equation,

$$\tan kL = (b_2/b_1) \cot kd - B/Y_{02}. \qquad (2)$$

The term b_2/b_1 results from the fact that it is equal to Y_{01}/Y_{02} as Marcuvitz [6] pointed out. Using the results in Fig. 5.26.3 in Marcuvitz [6], and $b_2/b_1 = 2$, (2) becomes

$$\tan kL = 2 \cot kd - 0.8(b_2/s)(1 + 2\Delta/\pi)k. \quad (3)$$

Equation (3) was solved for k using different values of the angle ϕ, and the factor η was calculated by (1). The solutions are shown in Table I along with Yashkin's results.

It is evident, therefore, that the two-step approximation for all practical uniform isosceles trapezoidal waveguides is sufficient to yield good results. Also, it is noted that only when the lateral walls are slanted considerably, i.e., large ϕ, is it necessary that the "discontinuity" admittances be taken into account. For $\phi = 60°$, the error in η is about 6 percent. This discrepancy is due to the fact that the B/Y_0 values used in these calculations are based on the assumption that the line is infinitely long on both sides of the discontinuity. Short circuits placed close to the discontinuity and other steps in close proximity will change the admittance [5]. Exact calculation of B may be made by Hahn's method [7].

Three-step approximations (see Table I) were developed, and the calculations yielded no significant improvement. Unlike the transverse resonance method, Yashkin's approximation technique gives more accurate results as the number of steps is increased. By the perturbation methods [8], it can be shown that the deviation from the exact cutoff wave number is reduced as the approximated boundaries become closer to the actual.

Although not shown here, results for the case $b/s = 0.50$ were even better than for $b/s = 0.25$, (where $b = b_2$ for two steps, and $b = b_3$ for three steps). Good results are expected to be obtained for guides with $b/s < 0.25$, provided that $A/s > 0.1$.

It is visualized that the transverse resonance method may be used for calculating the critical wavelengths of right-angled uniform trapezoids, and, many of the waveguide problems solved by Yashkin [2], [3].

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